ANNEX D

(to Recommendation E.506)

EXAMPLE OF A TOP DOWN MODELLING METHOD

The model for forecasting telephone traffic from Norway to the European countries is divided into two separate parts. The first step is an econometric model for the total traffic from Norway to Europe. Thereafter, we apply a model for the breakdown of the total traffic on each country.

D.1 *Econometric model of the total traffic from Norway to Europe*

With an econometric model we try to explain the development in telephone traffic, measured in charged minutes, as a function of the main explanatory variables. Because of the lack of data for some variables, such as tourism, these variables have had to be omitted in the model.

The general model may be written:

$$X_t = e^K \cdot GNP_t^a \cdot P_t^b \cdot A_t^c \cdot e^{ut} \quad (t = 1, 2, \ldots, N) \qquad \text{(D—1)}$$

where:

X_t is the demand for telephone traffic from Norway to Europe at time t (charged minutes).

GNP_t is the gross national product in Norway at time t (real prices).

P_t is the index of charges for traffic from Norway to Europe at time t (real prices).

A_t is the percentage direct—dialled telephone traffic from Norway to Europe (to take account of the effect of automation). For statistical reasons (i.e. impossibility of taking logarithm of zero) A_t goes from 1 to 2 instead of from 0 to 1.

K is the constant.

a is the elasticity with respect to *GNP*.

b is the price elasticity.

c is the elasticity with respect to automation.

u_t is the stochastic variable, summarizing the impact of those variables that are not explicitly introduced in the model and whose effects tend to compensate each other (expectation of $u_t = 0$ and var $u_t = \sigma^2$).

By applying regression analysis (OLSQ) we have arrived at the coefficients (elasticities) in the forecasting model for telephone traffic from Norway to Europe given in Table D—1/E.506 (in our calculations we have used data for the period 1951—1980).

The t statistics should be compared with the Student's Distribution with N — d degrees of freedom, where N is the number of observations and d is the number of estimated parameters. In this example, $N = 30$ and $d = 4$.

The model “explains” 99.7% of the variation in the demand for telephone traffic from Norway to Europe in the period 1951—1980.

From this logarithmic model it can be seen that:

— an increase in GNP of 1% causes an increase in the telephone traffic of 2.80%,

— an increase of 1% in the charges, measured in real prices, causes a decrease in the telephone traffic of 0.26%, and

— an increase of 1% in A_t causes an increase in the traffic of 0.29%.

We now use the expected future development in charges to Europe, in GNP, and in the future automation of traffic to Europe to forecast the development in telephone traffic from Norway to Europe from the equation:

$$X_t = e_t^{-16.095} \cdot GNP_t^{2.80} \cdot P_t u^{-0.26} \cdot A_t^{0.29} \qquad \text{(D—2)}$$

TABLE D—1/E.506

Coefficients	Estimated values	*t* statistics
K	—16.095	—4.2
a	2.799	8.2
b	— 0.264	—1.0
c	0.290	2.1

D.2 *Model for breakdown of the total traffic from Norway to Europe*

The method of breakdown is first to apply the trend to forecast the traffic to each country. However, we let the trend become less important the further into the period of forecast we are, i.e. we let the trend for each country converge to the increase in the total traffic to Europe. Secondly, the traffic to each country is adjusted up or down, by a percentage that is equal to all countries, so that the sum of the traffic to each country equals the forecasted total traffic to Europe from equation (D—2).

Mathematically, the breakdown model can be expressed as follows:

Calculation of the trend for country i:

$$R_{it} = b_i + a_i \cdot t, \quad i = 1, \ldots, 34 \quad t = 1, \ldots, N \qquad \text{(D—3)}$$

where

$R_{it} = \frac{X_{it}}{X_t}$, i.e country *i*'s share of the total traffic to Europe.

X_{it} is the traffic to country *i* at time *t*

X_t is the traffic to Europe at time *t*

t is the trend variable

a_i and b_i are two coefficients specific to country *i*; i.e. a_i is country *i*'s trend. The coefficients are estimated by using regression analysis, and we have based calculations on observed traffic for the period 1966—1980.

The forecasted shares for country *i* is then calculated by

$$R_{it} = R_{iN} + a_i \cdot (t - N) \cdot e^{-\frac{t-5}{40}} \qquad \text{(D—4)}$$

where *N* is the last year of observation, and e is the exponential function.

The factor $e^{\frac{t-5}{40}}$ is a correcting factor which ensures that the growth in the telephone traffic to each country will converge towards the growth of total traffic to Europe after the adjustment made in Equation (D—6).

To have the sum of the countries' shares equal one, it is necessary that

$$\sum_i R_{it} = 1 \qquad \text{(D—5)}$$

This we obtain by setting the adjusted share, $\overline{R}_{it}$, equal to

$$\overline{R}_{it} = R_{it} \frac{1}{\sum_i R_{it}} \qquad \text{(D—6)}$$

Each country's forecast traffic is then calculated by multiplying the total traffic to Europe, X_t, by each country's share of the total traffic:

$$X_{it} = \overline{R}_{it} \times X_t \qquad \text{(D—7)}$$

D.3 *Econometric model for telephone traffic from Norway to Central and South America, Africa, Asia, and Oceania.*

For telephone traffic from Norway to these continents we have used the same explanatory variables and estimated coefficients. Instead of gross national product, our analysis has shown that for the traffic to these continents the number of telephone stations within each continent are a better and more significant explanatory variable.

After using cross—section/time—series simultaneous estimation we have arrived at the coefficients in Table D-2/E.506 for the forecasting model for telephone traffic from Norway to these continents (for each continent we have based our calculations on data for the period 1961—1980):

TABLE D—2/E.506

Coefficients	Estimated values	*t* statistics
Charges	—1.930	—5.5
Telephone stations	2.009	4.2
Automation	0.5	—

We then have $R^2 = 0.96$. The model may be written:

$$X^k_t = e^K \cdot (TS^k_t)^{2.009} \cdot (P^k_t)^{1.930} \cdot (A^k_t)^{0.5} \qquad \text{(D—8)}$$

where

X^k_t is the telephone traffic to continent k (k = Central America, . . ., Oceania) at time t,

e^K is the constant specific to each continent. For telephone traffic from Norway to:

Central America: $K^1 = —11.025$

South America: $K^2 = —12.62$

Africa: $K^3 = —11.395$

Asia: $K^4 = —15.02$

Oceania: $K^5 = —13.194$

TS^k_t is the number of telephone stations within continent k at time t,

P^k_t is the index of charges, measured in real prices, to continent k at time t, and

A^k_t is the percentage direct—dialled telephone traffic to continent k.

Equation (D—8) is now used — together with the expected future development in charges to each continent, future development in telephone stations on each continent and future development in automation of telephone traffic from Norway to the continent — to forecast the future development in telephone traffic from Norway to the continent.

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