ANNEX B

(to Recommendation E.506)

EXAMPLE USING WEIGHTED LEAST SQUARES METHOD

B.1 *Telex data*

The telex traffic between the following countries has been analyzed:

— Germany (D)

— Denmark (DNK)

— USA (USA)

— Finland (FIN)

— Norway (NOR)

— Sweden (S)

The data consists of yearly observations from 1973 to 1984 [19].

B.2 *Forecasting*

Before using the weighted least squares method, separate forecasts for the traffic matrix have to be made. In this example a simple ARIMA (0,2,1) model with logarithmic transformed observations without explanatory variables is used for forecasting. It may be possible to develop better forecasting models for the telex traffic between the various countries. However the main point in this example only is to illustrate the use of the weighted least squares technique.

Forecasts for 1984 based on observations from 1973 to 1983 are given in Table B—1/E.506.

TABLE B—1/E.506

Forecasts for telex traffic between Germany(D), Denmark(DNK), USA(USA), Finland(FIN), Norway(NOR) and Sweden(S) in 1984

From / To	D	DNK	USA	FIN	NOR	S	Sum	Forecasted sum
D	—	4869	12 630	2879	2397	5230	28 005	27 788
DNK	5196	—	1655	751	1270	1959	10 831	10 805
USA	11 103	1313	—	719	1657	2401	17 193	17 009
FIN	2655	715	741	—	489	1896	6496	6458
NOR	2415	1255	1821	541	—	1548	7580	7597
S	4828	1821	2283	1798	1333	—	12 063	12 053
Sum	26 197	9973	19 130	6688	7146	13 034		
Forecasted sum	26 097	9967	19 353	6659	7110	12 914		

It should be noticed that there is no consistency between row and column sum forecasts and forecasts of the elements in the traffic matrix. For instance, the sum of forecasted outgoing telex traffic from Germany is 28 005, while the forecasted row sum is 27 788.

To adjust the forecasts to get consistency and to utilize both row/column forecasts and forecasts of the traffic elements the weighted least squares method is used.

B.3 *Adjustment of the traffic matrix forecasts*

To be able to use the weighted least squares method, the weights and the separate forecasts are needed as input. The separate forecasts are found in Table B—2/E.506, while the weights are based on the mean squared one step ahead forecasting errors.

Let y_t be the traffic at time t. The ARIMA (0,2,1) model with logarithmic transformed data is given by:

$$z_t = (1 - B)^2 \ln y_t = (1 - \theta B)\, a_t$$

or

$$z_t = a_t - \theta a_{t-1}$$

where

$z_t = \ln y_t - 2 \ln y_{t-1} + \ln y_{t-2}$

a_t is white noise,

θ is a parameter,

B is the backwards shift operator.

The mean squared one step ahead forecasting error of z_t is:

$$MSQ = \frac{1}{n} \Sigma (z_t - z\phi_{t-1}(1))^2$$

where

$\hat{z}_{t-1}(1)$ is the one step ahead forecast.

The results of using the weighted least squares method is found in Table B—3/E.506 and show that the factors in Table B—1/E.506 have been adjusted. In this example only minor changes have been performed because of the high conformity in the forecasts of row/column sums and traffic elements.

TABLE B—2/E.506

Inverse weights as mean as squared one step ahead forecasting errors of telex traffic (100^{-4}) between Germany(D), Denmark(DNK), USA(USA), Finland(FIN), Norway(NOR) and Sweden(S) in 1984

From / To	D	DNK	USA	FIN	NOR	S	Sum
D	—	28.72	13.18	11.40	8.29	44.61	7.77
DNK	5.91	—	43.14	18.28	39.99	18.40	10.61
USA	23.76	39.19	—	42.07	50.72	51.55	21.27
FIN	23.05	12.15	99.08	—	34.41	19.96	17.46
NOR	21.47	40.16	132.57	24.64	—	17.15	20.56
S	6.38	12.95	28.60	28.08	8.76	—	6.48
Sum	6.15	3.85	14.27	9.55	12.94	8.53	

TABLE B—3/E.506

Adjusted telex forecasts using the weighted least squares method

From / To	D	DNK	USA	FIN	NOR	S	Sum
D	—	4850	12 684	2858	2383	5090	27 865
DNK	5185	—	1674	750	1257	1959	10 825
USA	11 001	1321	—	717	1644	2407	17 090
FIN	2633	715	745	—	487	1891	6471
NOR	2402	1258	1870	540	—	1547	7617
S	4823	1817	2307	1788	1331	—	12 066
Sum	26 044	9961	19 280	6653	7102	12 894	